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Examples of pure research:

Constructivism – Piaget The magic number 7±2 – Miller Arithmetic bugs – Brown and Burton Behaviorism – Skinner Expertise - deGroot







In a design experiment, you have a theory of what will work, and why; empirical research refines both the theory and the intervention.

2. The question of values

We are often asked questions like,

- Are large classes as good as small classes?
- Is curriculum A better than Curriculum B?
- What is the right way to teach X?

These questions seem to make sense, but they are unanswerable. The answers all depend on <u>values</u> - on what you consider to be the most important outcomes.

Consider the question, Are large math classes as good as small classes?" How will you judge the outcomes? Will you look at: - Scores on a multiple-choice final exam? - Skills at modeling and problem solving? - How many students continue studying mathematics? The answer to "what is best" depends on what you value - and how you measure it.



3a. A Framework for Conceptualizing the Research Process

The next slide shows how most people (including mathematicians and scientists) tend think of the representation and modeling process.



















- a. -3.5, .75, 1.5, 4.5, -.75, -2.5, 4.75, 2.75, .5, -1.5, 2.25, 9.25, 3.5, 1.25, -.5, 2.5, .5, 7.25, 5.5, 3;
- b. 3.75, 4.5, 3, 5, 2.25, 1.25, .75, 3, -.5, 1.5, 3.5, 6, 4.5, 5.5, 2.5, 4.25, 2.75, 3.75, 4.75







You work for a business that has been using two taxicab companies, Company A and Company B.

Your boss gives you a list of (early and late) "Arrival times" for taxicabs from both companies over the past month.

Your job is to analyze those data using charts, diagrams, graphs, or whatever seems best. You are to:

i. make the best argument that you can in favor of Company A;

ii. make the best argument that you can in favor of Company B;

iii. write a memorandum to your boss that makes a reasoned case for choosing one company or the other, using the relevant mathematical tools at your disposal.

	Company A		В	
3 mins 30 secs	Early	3 mins 45 secs	Late	
45 secs	Late	4 mins 30 secs	Late	
1 min 30 secs	Late	3 mins	Late	
4 mins 30 secs	Late	5 mins	Late	
45 secs	Early	2 mins 15 secs	Late	
2 mins 30 secs	Early	2 mins 30 secs	Late	
4 mins 45 secs	Late	1 min 15 secs	Late	
2 mins 45 secs	Late	45 secs	Late	
30 secs	Late	3 mins	Late	
1 minute 30 secs	Early	30 secs	Early	
2 mins 15 secs	Late	1 min 30 secs	Late	1
9 mins 15 secs	Late	3 mins 30 secs	Late	
3 mins 30 secs	Late	6 mins	Late	
1 min 15 secs	Late	4 mins 30 secs	Late	
30 secs	Early	5 mins 30 secs	Late	
2 mins 30 secs	Late	2 mins 30 secs	Late	
30 secs	Late	4 mins 15 secs	Late	
7 mins 15 secs	Late	2 mins 45 secs	Late	
5 mins 30 secs	Late	3 mins 45 secs	Late	
3 mins	Late	4 mins 45 secs	Late	

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Same Data:	Distribution A	Distribution B	
Mean	2.3	<u>3.14</u>	
Median	1.53	3.15	
Range	12.45	6.3	
SD	3.11	1.4	



















A superintendent did a study where he compared three "experimental" schools with three "control" schools, and got no differences on average.

One of the schools worked with the program and got great results, one was so-so, and one resisted the program and got lousy results.

What's the moral of the story?







Accountability to data.

As a field, we are much too casual in our use of data, especially qualitative data. We tend to formulate our ideas about what is important, and then offer segments of data that substantiate our ideas ("confirmation bias").

We must gather data that allows for the possibility of proving ourselves wrong, and we must find ways to deal comprehensively with all the data we gather.

There is also the matter of how we hold ourselves accountable. My own personal standard for qualitative data is to be able to model people's actions and decision-making. The reason is that such modeling guards against ad hoc justifications, and forces me to try to explain everything, or explain why I can't.

From my personal perspective, model-building as a form of theory testing is essential.





Aspects of trustworthiness include:

- Descriptive and explanatory power
- Prediction and falsification
- Rigor and specificity
- Replicability
- Triangulation

In fact, our whole "tour" of the diagram was devoted to issues of trustworthiness.

Generality

Beware of claims about generality – there are different types:

The *claimed generality* of a body of research is the set of circumstances in which the author of that work claims that the findings of the research apply.

The *implied generality* of the work is the set of circumstances in which the authors of that work appear to suggest that the findings of the research apply.

The *potential generality* of the work is the set of circumstances in which the results of the research might reasonably be expected to apply.

The *warranted generality* of the work is the set of circumstances for which the authors have provided trustworthy evidence that the findings do apply.

Importance

(I shouldn't have to say much here!)

Studies should work toward the purposes discussed at the beginning of this talk:

Pure (Basic Science) – To understand the nature of mathematical thinking, teaching, and learning; and

Applied (Engineering) – To use such understandings to improve mathematics instruction.



There are various forms and methods of rigorous research.

These may correspond to:

- Large-scale statistical studies (properly conducted and interpreted)
- Individual findings correlated to large studies
- Replications over time
- · Detailed analytic models tied to theory
- Predictions tied to theory and models



Individual findings correlated to large studies, for example:

My studies of student beliefs in one classroom, backed up by questionnaires given to 240 students, which demonstrate they are typical, or triangulated with results on the US National Assessment of Educational Progress.

Replications over time:

- Thousands of students with the same "read/explore" pattern as in the graphs I showed in my talk on problem solving. (More than 50% of all students, over many years.)
- It's easy to replicate Miller's "magic number 7±2". Similarly for perception of illusions, vertical translations, etc.



Our responsibility is to use appropriate methods – often multiple methods – for the issues we are investigating...

... and then make careful claims that to not go beyond what the results indicate. (Though we can always suggest what we think will be true.)

In Summary...

- Education is fundamentally concerned with theories of thinking, teaching, and learning.
- Education is fundamentally concerned with the improvement of educational practice.
- Like any empirical discipline, it profits from the exchange between theory and practice.
- As an empirical discipline, it adheres to the standards for the empirical sciences. We must do rigorous work, with well established norms and standards.

